HAW HAMBURG

Report for NASA/DLR Design Challenge 2018

Design of an Ultra Efficient Passenger Aircraft

Task 4:
Aircraft Design -
An Approach with Breguet Range Equation

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Abstract

There is an increasing public, political and economic demand for reducing the fuel consumption for aircrafts. In order to meet this interest, this evaluation will present how Aircraft Design can make a contribution to this request.
The parameters for aircraft performance depends mainly on the aerodynamics, propulsion system and the structure. All essential parameters will be considered in the Breguet range equation: the glide ratio (lift-to-drag ratio), the specific fuel consumption and the mass fraction. The Breguet range equation is discussed under the aspect of fuel consumption with the according fuel calculation. A detailed drag estimation method is derived for aircraft parameters such as glide ratio, wing loading and aspect ratio.
This report will indicate, how the Breguet-range equation can be used as a simple but significant model for the fuel consumption on transportation aircrafts.
An example mission with a payload-range diagram for a midrange aircraft (like Airbus A320 or Boeing 737) is used to demonstrate the efficiency of this approach.
Results will be verified with a constraint diagram and payload-range diagram with a preliminary design process.

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List of Symbols

a	Speed of sound
a	Take-off slope
A	Aspect ratio
b	Wing span
BPR	Bypass ratio
B_S	Breguet range factor
B_t	Breguet time factor
R	Range
R´	Derivation of range R
c	Specific Fuel Flow SFC
c_P	Power Specific Fuel Flow PSFC for propeller aircraft
c_T	Thrust Specific Fuel Flow TSFC for jet aircraft
C_D	Drag coefficient
$C_{D,0}$	Zero drag coefficient
$C_{D,i}$	Induced drag coefficient
$C_{D,int}$	Interference drag coefficient
$C_{D,W}$	Wave drag coefficient
C_L	Lift coefficient
$C_{L,max,L}$	Maximum lift coefficient in landing configuration
$C_{L,max,TO}$	Maximum lift coefficient in take-off configuration
D	Drag
D_i	Induced drag
D_{int}	Interference drag
D_w	Wave drag
D_0	Zero lift drag
e	Oswald factor
E	Glide ratio, L-over-D
E_{max}	Maximum glide ratio
E_{TO}	Take-off glide ratio
E_L	Landing glide ratio
E_{TO}	Take-off glide ratio
E	Energy or work W
E´	Mass specific energy of the battery [Wh/kg] or the heating value H [J/kg] of fuel
g	Gravitation constant on earth
H	Heating value
k_{APP}	Approach factor
k_L	Landing factor
k_{TO}	Take-off Field Length factor
L	Lift

L/D	L-over-D, glide ratio E
m	Mass
m_{bat}	Battery mass
m_{ac}	Constant aircraft mass
m_{ave}	Average aircraft mass
m_{Cr}	Cruise mass
m_{ff}	Fuel ratio
m_F	Fuel mass
m_{final}	Final mass m_2
m_{intial}	Initial mass m_1
m_{Ldg}	Landing mass
m_{ML}	Maximun Landing mass
m_{MPL}	Maximum Payload mass
m_{MTO}	Maximum Take-off mass
m_{OE}	Operating Empty mass
m_{PL}	Payload mass
m_{Res}	Fuel reserve mass
m_{TO}	Take-off mass
M_{Cr}	Cruise Mach number
M_{ff}	Total Fuel Fraction
$M_{ff,i}$	Fuel Fraction of mission segment i
n_E	Number of engines
P	Power
P	Pressure
P_{bat}	Battery power
Q	Fuel flow
R	Range
R_{Cr}	Cruise range
R_D	Design range
R_{Ferry}	Ferry Range, max. range with no payload
R_{MF}	Maximum range with full tank and payload
R_{MPL}	Maximum range with maximum payload
S_w	Wing area
S_{LFL}	Landing Field Length
S_{TOFL}	Take-off Field Length
t	Endurance, time
T	Thrust
T_{TO}	Take-off thrust
T_{Cr}	Cruise thrust
T/W	Aircraft Thrust-to-Weight ratio
v	Velocity, speed
v_{Cr}	Cruise speed

v_{APP}	Approach speed
V	Volume
W	Weight m·g

Greek Symbols

γ	Angle of climb
γ	Ratio of specific air heats
ρ	Density
σ	Relative Density
η	Efficiency
$\eta_{overall}$	Overall Efficiency
η_{prop}	Propeller Efficiency
η_{shaft}	Shaft Efficiency
ϑ	Range ratio factor

List of Abbreviations

CeRAS	Central Reference Aircraft Data System
DLR	Deutsches Zentrum für Luft- & Raumfahrt
ISA	ICAO Standard Atmosphere
MTOW	Maximum Take-Off Weight
MZF	Maximum Zero Fuel Weight
NACA	National Advisory Committee for Aeronautics
NASA	National Aeronautics and Space Administration
OEW	Operating Empty Weight
PAX	Passenger
PSFC	Power Specific Fuel Consumption
PreSTo	Preliminary Sizing Tool
SAR	Specific Air Range
SFC	Specific Fuel Consumption
TSFC	Thrust Specific Fuel Consumption
TLAR	Top Level Aircraft Requirement

List of Definitions

Breguet
Louis Charles Breguet was a 1880 born aircraft designer, who is falsely considered as the originator of the “Breguet Range Equation”. Originally, this equation was introduced in 1920 by J. G. Coffin in his NACA Report. Since this equation is known as the Breguet Range Equation, it will be called in this project Breguet Range Equation as well.

CeRAS
CeRAS (Central Reference Aircraft data System) is a central database hosting reference design data of commercial aircraft. It is intended to be used for research projects dealing with conceptual to preliminary aircraft design studies as well as technology integration and assessment.

PreSTo
PreSTo (Preliminary Sizing Tool) is an Excel spreadsheet developed at the HAW by Scholz 2017 for preliminary aircraft design sizing.

1 Introduction

The purpose of this document is to provide an overview about aircraft design process with the help of the Breguet range equations. The Breguet range equations for 4 different aircraft types (turbojet, turboprop, battery and hybrid driven) are derivated. The remaining part of this document presents a detailed application of the Breguet range equation for preliminary sizing in aircraft design especially under the aspect of fuel consumption. Data for an Airbus A320 are used to redesign an aircraft with the Excel spreadsheet PreSTo. A practical approach for calculating aircraft parameter are given. Results are illustrated with the payload-range diagram. For a better overview the conceptual aircraft design as well as a detailed lift and drag estimation is outside of the scope of this document and will be handled in a separate document.

1.1 Motivation

The main motivation for this report is to supply an overview and understanding of aircraft design processes for students who want to participate in the NASA/DLR Design Challenge 2018 (Hartman 2018) in order to design an ultra efficient passenger aircraft of the future. For a subsonic aircraft NASA push new technology that will support development of new aircraft products that meet the long-term goals (beyond year 2035) like 80% reduction in noise , 80% reduction in NO_x emission for take-off, landing and cruise and 60-80% reduction for the fuel/energy consumption (NASA 2017) relative to the best in class aircraft in 2005.

1.2 Structure

Chapter 1: Introduction

Chapter 2: Derivation of 4 different Breguet range equations for turbojet, turboprop, battery and hybrid (new equation) driven aircraft.

Chapter 3: Estimation of the burned fuel mass with the Breguet range factor, the definition of the constant specific fuel consumption SFC and calculation of the specific air range SAR as a function of the Breguet range equation and derivated from the payload range diagram.

Chapter 4: Estimation of the mass fuel fraction for the cruise segment with the help of the payload-range diagram and the Breguet range equation.

Chapter 5: New structured aircraft design. Preliminary sizing with PreSTo of a reference aircraft Airbus A320 with CeRAS data and the meaning and correlation of the Breguet range equation for aircraft design.

Chapter 6: Discussion and Conclusion

2 Aircraft Design and Breguet Range Equation

2.1 The Relevance of Breguet Range Equation

The most important parameters in aircraft design are related to drag, lift, weight and the propulsion system. All these parameters have an impact on the range performance of the aircraft. An improvement of this parameter such as the reduction of the drag have a direct impact to the range and therefore for the fuel consumption of the aircraft.
The Breguet range equation is not a simple conversion of the well known velocity formular with R = v*t (range = speed x time) as in this case the range R simply depends on the velocity v and the time t. or R=f(v,t). In aircraft design a more detailed range equation which depends on the above mentioned parameters (drag, lift, weight and fuel consumption) is useful.

2.1.1 Derivation of the Breguet Range Equation

With reference to Young 2001 the Breguet range equation is derivated as follows:
Starting with the Specific Air Range (SAR) defined as the distance per unit consumed fuel mass.

$$\mathrm{SAR} = -\frac{dx}{dm_F} \qquad (2.1)$$

where x is the air range and m the consumed burned fuel mass. As the fuel mass is reducing during the flight, it has a negative sign. The units of SAR are [km/kg]. In the European automotive industry the more familiar inverted value 1/SAR is used with the units [kg/km] or with the known specific density ρ the volume in liter can be calculated with the units [l/km] for 1/SAR.

The fuel flow Q (rate of burnt fuel) is defined as the burned fuel mass m_F per time *t*:

$$\mathrm{Q} = -\frac{dm_F}{dt} \qquad (2.2)$$

With the true air speed (TAS) $\mathrm{V} = \frac{dx}{dt}$ the definition of SAR can now rewrite as

$$SAR = \frac{V}{Q} \qquad (2.3)$$

In a level flight thrust is equal to the drag (T=D) and the lift is equal to the weight of the aircraft (L=W). With the definition of the Specific Fuel Consumption (SFC) as the fuel flow Q per unit thrust

$$c_T = \frac{Q}{T} \tag{2.4}$$

For a jet aircraft in level flight the fuel flow Q can now be written as:

$$Q = c_T \cdot T = c_T \cdot D = c_T \left(\frac{D}{L}\right) \cdot \mathrm{W} = \frac{c_T}{E} \cdot m \cdot g$$

The air range is given by

$$R = -\int_{m_1}^{m_2} \frac{V}{Q} \mathrm{d}m = \int_{m_1}^{m_2} \frac{V \cdot E}{C_T \cdot g} \cdot \frac{1}{m} \mathrm{d}m \tag{2.5}$$

As the airspeed V and the lift coefficient C_L (and therefore the glide ratio E) are constant, the result of the integration is

$$R = \frac{V \cdot E}{C_T \cdot g} ln\left(\frac{m_1}{m_2}\right) \tag{2.3}$$

2.2 Breguet Range Equation for Jet Aircraft

Equation (2.6) is known as Breguet Range Equation for jet engines. With the glide ratio $E=L/D=C_{L/}C_D$ ("L-over-D") and m_1 as the initial mass (not fuel mass m_F !) of the aircraft at Take-Off and m_2 as the final mass of the aircraft after landing. Thus the Breguet Range Equation can also be written as:

$$R = \frac{L}{D} \frac{v}{TSFC \cdot g} \ln\left(\frac{m_{initial}}{m_{final}}\right) \tag{2.4}$$

where:

R [m] is the air range of the flying distance
$E = L/D = C_{L/}C_D$ [-] is the glide ratio or L-over-D
V [m/s] is the true airspeed (TAS) of the aircraft
$m_1 = m_{initial}$ [kg] is the initial mass of the aircraft
$m_2 = m_{final}$ [kg] is the final mass of the aircraft
$m_F = m_1 - m_2 = m_{initial} - m_{final}$ [kg] mass of the consumed fuel for flying the air range R
c_T = TSFC = [kg/N/s] thrust specific fuel consumption for a jet engine
g [m/s^2] is the earth gravitation constant (~9.81m/s^2)
ln is the natural logarithm with the Euler number e = 2.71828.. as base value

2.3 Breguet Range Equation for Propeller driven Aircraft

As a propeller driven aircraft will supply power instead of thrust, we simply have to exchange the thrust T by power P in the above Breguet Range Equation.

From mechanic we know that the power P is achieved by multiplying the thrust T by the velocity V so that $P = T \cdot V$. Furthermore it is known that the shaft power from the engine will not 100% transferred to propeller power, we have to consider accordingly the propeller efficiency. Hence:

$$P = T \cdot \mathrm{V} \cdot \eta_P \tag{2.5}$$

where:

P [kW] available power at the propeller ($P = P_{shaft} \cdot \eta_P$)
T [N] thrust
η_P [-] propeller efficiency

From the Breguet Range Equation (2.6) for jets, the air range for a propeller driven aircraft is now given by:

$$R = \frac{L}{D} \frac{\eta_P}{c_P \cdot g} ln\left(\frac{m_1}{m_2}\right) \tag{2.6}$$

where:

η_P [-] propeller efficiency
c_P = PSFC [kg/kW/s] power specific fuel consumption for a propeller aircraft

This equation is the Breguet Range Equation for a propeller driven aircraft.
Similar to Eqn. (2.6) E (and therefore C_L), c_P (and therefore v) and η_P are constant.

The specific fuel consumption c_P is now related to the power and has therefore a different unit comparing with the thrust specific parameter c_T from Eqn. (2.6). Both parameters are related by the airspeed v only:

$$c_T = c_P \cdot \mathrm{v} \tag{2.7}$$

2.3.1 Breguet Range Equation as a Function of Energy Storage System

If the power specific fuel consumption c_P is unknown, we are able to calculate the parameter with the heating value H [J/kg] of the fuel. As we will see in the next chapter the units of the heating value H are the same as for the energy density of batteries E'[Ws/kg], so H = E'.

With the relation

$$c_P = \frac{1}{E\prime} = \frac{1}{H}\left[\frac{kg}{W\cdot s}\right] \qquad (2.8)$$

and the overall efficieny $\eta_{overall}$ of the above Breguet Range Equation is thus defined as a function of the heating value or of the energy density by:

$$R = \frac{L}{D}\frac{E\prime\cdot\eta_{overall}}{g} ln\left(\frac{m_1}{m_2}\right) \qquad (2.9)$$

A typical value for kerosene is E'=43 MJ/kg or divided by 3600 s we get E= 11944 Wh/kg.

The charme of this equation is, that it can compare or combine a battery powered aircraft with a fuel powered aircraft. But we have to consider a different overall efficieny $\eta_{overall}$
which is approximately 2 times better for batteries than for fuel (due to the lower thermal efficiency ~40%).

2.4 Breguet Range Equation for Battery powered Aircraft

For using the above Breguet Range Eqns. (2.6) and (2.8) it is essential to know how the equation is derivated. It will not work for an aircraft with no mass reduction due to the burned fuel. A battery or solar powered engine has no reduction in the fuel mass m_F as the weight of the batteries or the solar panels as a power source is not changing. For $m_1 = m_2$ as masses at the begin and at the end of the flight $\ln(m_1/m_2) = \ln 1 = 0$ the result of Equations (2.6) and (2.8) is always R = 0.

Let us start again with the basic range equation $R = v\cdot t$. From mechanic we know that the unit of work W and energy E is the same: Watt [W]. Power is defined as work or energy per time:

$$P = \frac{W}{t} = \frac{E}{t} \qquad (2.10)$$

where:

P [kW] is the aircraft power
E [kWh] is work W or the energy of the battery with $W=F\cdot s$ [Nm] and $F=m\cdot a$ [N]
t [s] the flight time

With the definition of the mass specific energy

$$E' = \frac{E}{m} \qquad (2.11)$$

as the energy per mass [kWh/kg]. Inserting into the above equations the range R is defined by

$$R = v \cdot E' \cdot m_{bat}/P_{bat} \qquad (2.12)$$

In order to rearrange R as a function of the glide ratio L/D we use the power definition and for horizontal flight the constraint L=W and D=T

$$P = D \cdot v = \frac{m \cdot g}{L/D} \cdot v \qquad (2.13)$$

and the overall efficiency $\eta_{overall}$ between the battery power and the aircraft propeller power

$$P = P_{bat} \cdot \eta_{overall} \qquad (2.14)$$

where:

P [kW] is the aircraft power at the propeller

$\eta_{overall}$ [-] is the overall efficiency from the battery via the shaft to the propeller

$$\eta_{overall} = \eta_{shaft} \cdot \eta_{prop}$$

Finally the range equation for battery powered aircraft according Hepperle 2013 is

$$R = \frac{L}{D} \cdot \frac{E' \cdot \eta_{overall}}{g} \cdot \frac{m_{bat}}{m_{ac}} \qquad (2.15)$$

where:

E' [Wh/kg] mass specific energy of the battery or the heating value H [J/kg] of fuel

m_{bat} [kg] constant mass of the battery

m_{ac} [kg] constant mass of the aircraft, $m_{ac} = m_{TO} = m_{Ldg}$

Like the Breguet range equation for propeller driven engine the airspeed v is not part of the equation but it will indirectly affect the range R via the glide ratio L/D and the overall efficiency factor $\eta_{overall}$.

The actual highest available mass specific energy (energy density) for batteries like Lithium-Polymer (LiPo) batteries, have an energy density of E'_{bat}= 240 Wh/kg and is not improving sufficiently over the last decade.for a full use in a midsize aircrafts (Pellenwessel 2012). Comparing with fuel like kerosene with a typical energy density of E'=43 MJ/kg or divided by 3600 we get

$E'_{Kerosene}$= 11944 Wh/kg. This means that the energy density of fuel is approximately 50 times higher or in other words you need a 50 times higher mass for batteries than for kerosene fuel to supply the same power for the engine. But the overall efficiency $\eta_{overall}$ with batteries (with an electric motor) as power source is about **2 times** better than for fuel (with a combustion engines).

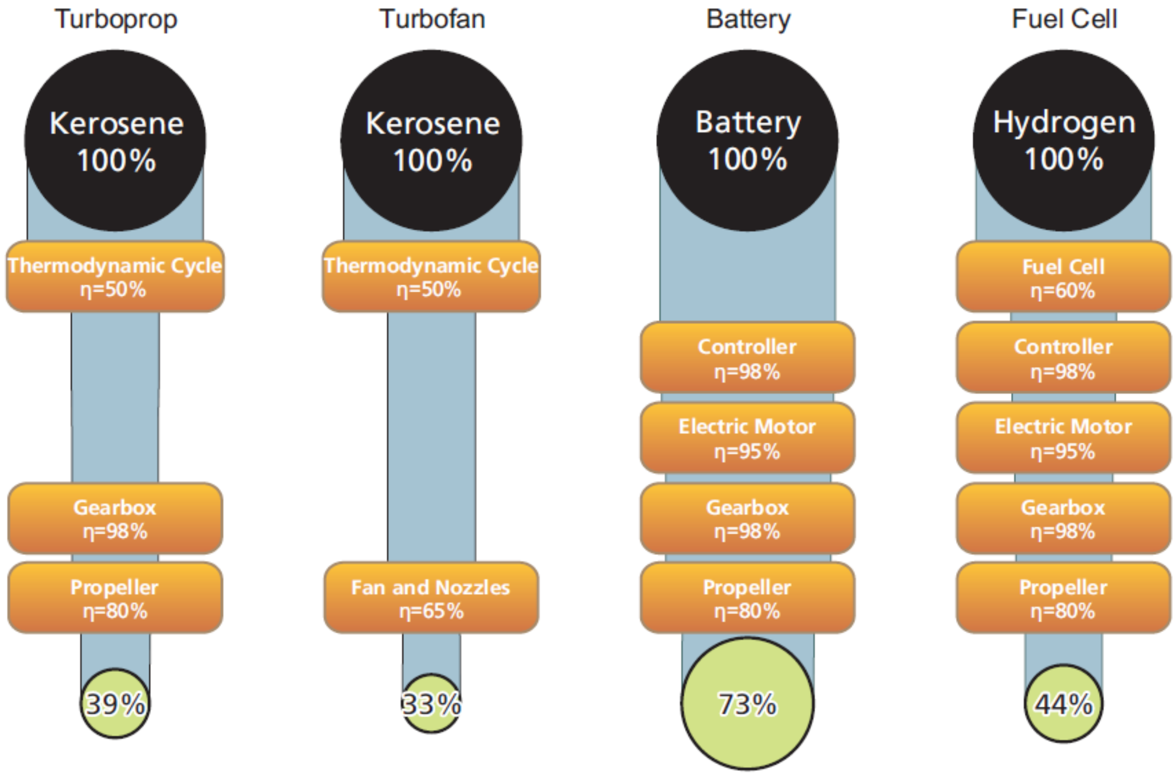


Figure 2.1: Component efficiency and overall efficiency acc. Hepperle 2013

With the actual (6/2018) state of technical readiness we still need about 25 time more battery mass m_{bat} than fuel mass m_F:

$$m_{bat} \sim 25 \cdot m_F$$

This lead also to a 25 times higher mass fraction for batteries m_{bat}/m than for fuel m_F/m_{TO}.

2.5 Breguet Range Equation for Hybrid powered Aircraft

Due to the before mentioned reason (25 times higher mass fraction for batteries) a hybrid powered aircraft, a combination of batteries and fuel as power source is the best of both world and seems to makes more sense for a mid range aircraft.

With the Breguet equation (2.x) and (2.y) for fuel and battery powered aircraft and the definition of the fuel fraction M_{ff} for a total flight mission

$$M_{ff} = \frac{m_{final}}{m_{initial}} = \frac{m_2}{m_1} \qquad (2.16)$$

Hence:

$$\frac{1}{M_{ff}} = \frac{m_{initial}}{m_{final}} = \frac{m_1}{m_2} = \frac{1}{1 - m_F/m_{TO}} \qquad (2.17)$$

where:

M_{ff} [-] is a ratio, the fuel fraction for the total flight
m_{final} [kg] is the aircraft mass at the end of the flight mission
$m_{initial}$ [kg] is the aircraft mass at the begin of the flight mission

the redefined Breguet range equations are given now for a propeller driven Aircraft with fuel as power source by:

$$R = \frac{E \cdot \eta_P \cdot \eta_F}{c_P \cdot g} ln\left(\frac{1}{1 - m_F/m_{TO}}\right) \qquad (2.18)$$

and for a propeller driven aircraft with batteries as power source by:

$$R = \frac{L}{D} \cdot \frac{E' \cdot \eta_P \cdot \eta_{bat}}{g} \cdot \frac{m_{bat}}{m_{TO}} \qquad (2.19)$$

We have to note that the overall efficiency of both aircraft systems are different with the before mentioned factor 2.

If we now decide, that the batteries are only in charge for a certain percentage ϑ of the total range R

$$R = (1 - \vartheta) \cdot R + \vartheta \cdot R \qquad (2.20)$$

we only have to insert the ratio factor $\vartheta = R_{bat} / R$ and adding both above equations to get the new Breguet range equation for a hybrid electric aircraft:

$$R = (1 - \vartheta) \left[\frac{L}{D} \cdot \frac{\eta_P \cdot \eta_F}{c_P \cdot g} ln\left(\frac{1}{1 - \frac{m_F}{m_{TO}}}\right)\right] + \vartheta \left[\frac{L}{D} \cdot \frac{E' \cdot \eta_P \cdot \eta_{bat}}{g} \cdot \frac{m_{bat}}{m_{TO}}\right] \qquad (2.21)$$

One of the current famous future project is the development of a hybrid-electric flight demonstrator E-FanX , a partnership of Airbus, Rolls-Royce and Siemens (www.airbus.com)

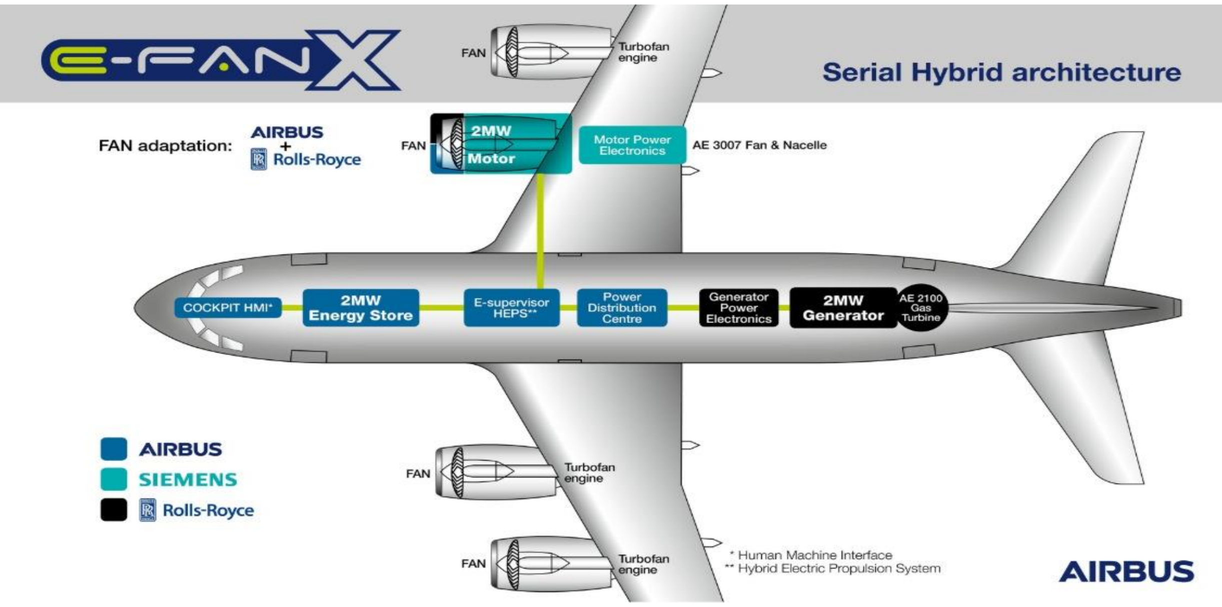


Figure 2.2 Architecture of the hybrid-electric aircraft E-Fan X

2.6 Breguet Range Equations Overview

Table 2.1 Breguet Range Equation for different aircraft propulsion systems

Aircraft type	Breguet Range Equation	Constants
Jet	$R = \frac{L}{D} \cdot \frac{V}{C_T \cdot g} ln\left(\frac{m_1}{m_2}\right)$	V, C_L, C_D, C_T h ≠ const.
Propeller	$R = \frac{L}{D} \cdot \frac{\eta_P}{c_P \cdot g} ln\left(\frac{m_1}{m_2}\right)$ or $R = \frac{L}{D} \cdot \frac{E' \cdot \eta_P \cdot \eta_F}{g} ln\left(\frac{m_1}{m_2}\right)$	V, C_L, C_D, C_P h ≠ const. E' = H (energy density = heating value)
Battery	$R = \frac{L}{D} \cdot \frac{E' \cdot \eta_P \cdot \eta_{bat}}{g} \cdot \frac{m_{bat}}{m_{TO}}$	V, C_L, C_D, E' m_{bat}, m_{TO}
Hybrid	$R = (1-\vartheta)\left[\frac{L}{D} \cdot \frac{\eta_P \cdot \eta_F}{c_P \cdot g} ln\left(\frac{1}{1-\frac{m_F}{m_{TO}}}\right)\right] + \vartheta\left[\frac{L}{D} \cdot \frac{E' \cdot \eta_P \cdot \eta_{bat}}{g} \cdot \frac{m_{bat}}{m_{TO}}\right]$	V, C_L, C_D, E', C_P

3 Fuel Consumption

3.1 Consumed Fuel Mass

To calculate the specific fuel consumption we reorganize the classical Breguet range equation as follows:

$$C_T = \frac{L}{D} \cdot \frac{V}{R \cdot g} ln\left(\frac{m_1}{m_2}\right) \tag{3.1}$$

The constant part of this equation is called the Breguet range factor with the unit [m]:

$$B_S = \frac{L}{D} \cdot \frac{V}{R \cdot g} = const. \tag{3.2}$$

Thus

$$R = B_S \cdot ln\left(\frac{m_1}{m_2}\right) \tag{3.3}$$

With the burned fuel mass $m_F = m_1 - m_2$ the range is given by:

$$R = B_S \cdot ln\left(\frac{m_2 + m_F}{m_2}\right) = B_S \cdot ln\left(1 + \frac{m_F}{m_2}\right) \tag{3.4}$$

or rearranged the mass m_1 as a linear function of the range m_1=f(R):

$$\frac{m_1}{m_2} = e^{\frac{R}{B_S}} \tag{3.5}$$

or:

$$m_F = m_2 \cdot (e^{\frac{R}{B_S}} - 1) \tag{3.6}$$

where:

m_F [kg] is the consumed fuel for the range R
$m_2 = m_{final}$ [kg] is the aircraft mass at the end of the trip
B_S [m] is the constant Breguet factor.

3.2 Specific Fuel Consumption SFC

The thrust specific fuel consumption is assumed as a constant value for a horizontal flight. It is a specific value, meaning relatet to 1 Newton and not to the used thrust T_{Cr} [N] for the airspeed V_{Cr} in cruise. The thrust T_{Cr} depends beside others on the aircraft weight [N], which will be reduced during the flight due to the burned fuel. So, the specific fuel consumption remains constant (SFC = const.) during a level flight but the fuel consumption expressed by the specific air range SAR [kg/m] will not (SAR ≠ const.).

A typical value for an engine of an Airbus A320 with a cruise speed M_{Cr}=0.76 => v=244 m/s at an altitude of 11000m is:

$c_T = 1.66 \cdot 10^{-5} \left[\frac{kg}{Ns}\right]$ and the same aircraft with a propeller propulsion: $c_P = 6.8 \cdot 10^{-8} \left[\frac{kg}{Nm}\right]$

Example: To get a better feeling for this numbers, a full powered A320 with a 120 kN engine is consuming every second ~2 kg (=$1.66 \cdot 120000 \cdot 10^{-5}$) which are ~2.5 liter fuel. So, for 100 km flight range (in 410 seconds) and 150 Pax on board the content meaning of

$c_T = 1.66 \cdot 10^{-5} \left[\frac{kg}{Ns}\right]$ is equal to 6.8 l/100km/Pax.

3.3 Specific Air Range SAR

The specific air range (SAR) is defined as the fuel mass reduction dm_F for a certain flight range dR or as the fuel volume related to the fuel flow Q (rate of burnt fuel per time):

$$SAR = \frac{dR}{dm} = \frac{V}{Q} \left[\frac{m}{kg}\right] \tag{3.7}$$

Example: An Airbus A320 with a fuel consuming of dm_F=14360 kg =17950 liter for the flight distance dR=2500 NM=4630 km has a specific air range of SAR=0,258 km/kg.
For a 1km flight distance the engines are consuming about 4 kg fuel. To make this value comparable with the SFC, the SAR for a 100 km trip is 400 kg=500 liter kerosene and with 150 Pax on board the meaning of

$\mathrm{SAR} = 0.258\ \mathrm{km/kg}$ is equal to 3.3 l/100km/Pax.

Due to the fuel mass reduction during the flight, the aircraft weight is reduced and therefore the specific air range SAR. Compared to the SFC value SAR is not constant for the whole flight distance but it is reducing:

$$\text{SAR} \neq \text{const.}$$

$$\text{SFC} = \text{const.}$$

The Specific Air Range is direct readable as a slope from the Payload-Range-Diagram (see next chapter) with the consumed fuel mass dm and the according distance dR between the known coordinates (R_i/m_i) of 2 points.
But to be mathematicaly correct, the specific air range is the first derivative of the Breguet Range Equation with respect to the mass m and can be calculated for a certain flight distance:

$$SAR = R' \tag{3.8}$$

In the below graphic the specific air range is the gradient angle of the tangent of the Breguet range equation at a certain range point R_i is shown:

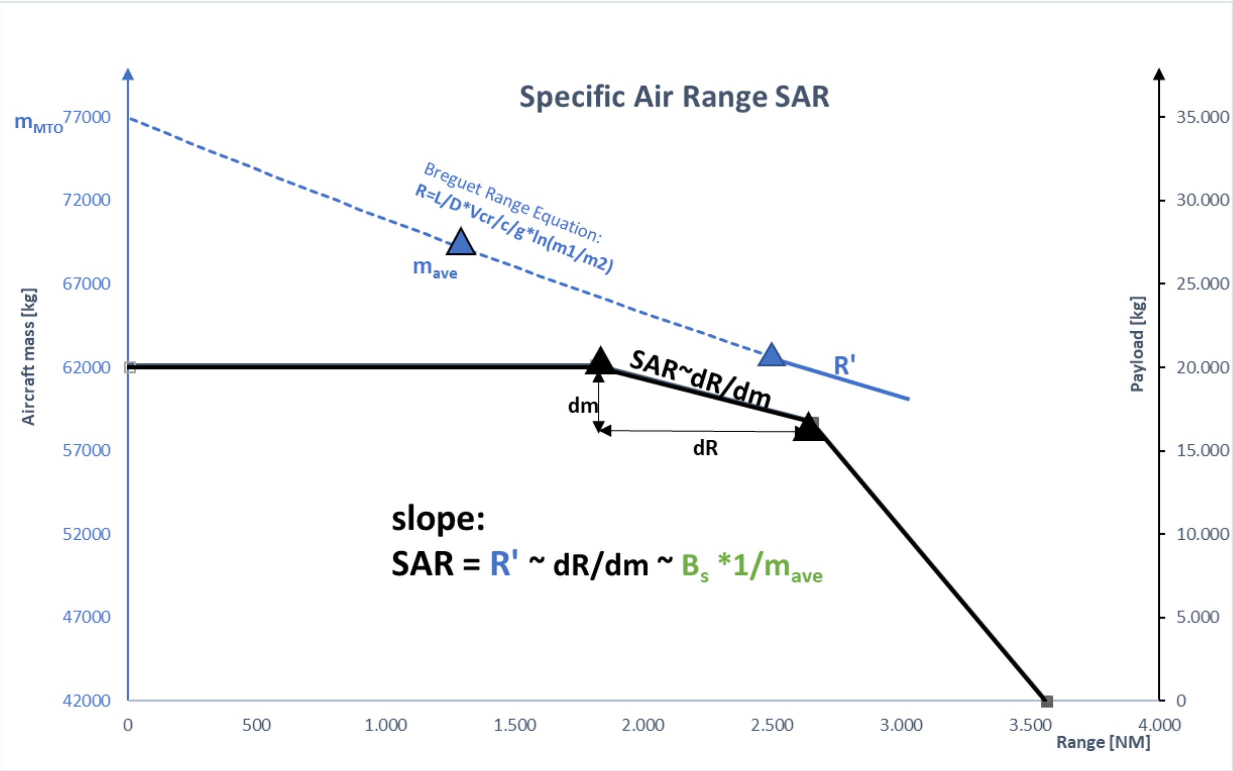


Figure 3.1 Specific Air Range SAR in Payload-Range Diagram

A very good approximate soluion for the range derivation R' is gained with the introduction of the average aircraft mass $m_{ave}=(m_1-m_2)/2+m_2$ for the flight:

$$SAR = B_s \cdot \frac{1}{m_{ave}} \tag{3.9}$$

Now we have different methods and approximate solutions with different accuracies to compute the specific air range SAR:

$$SAR = \frac{V}{Q} = R' = \frac{dR}{dm} = B_S \cdot \frac{1}{m_{ave}} \qquad (3.10)$$

Even with the rule of proportion (if we consider the m_1-line and m_2-curve in Fig. 3.1 as a triangle) we achieve an accuracy of less than 2.2 %. The mathematical exact solution is to compute SAR with the derivation R' but if only graphical information like the payload-range diagram is available than the approximation with the slope (dR/dm) or with the average mass m_{ave} are sufficient.

4 Estimation of the Fuel Fraction with the Payload-Range Diagram

4.1 Payload Range Diagram

The payload range diagram illustrates the relation between the payload m_{PL} (pax and cargo mass but excluding fuel mass) and the flight range R of the aircraft.

Once reached the maximum payload m_{MPL} the additional fuel mass is limited to the MTOW m_{MTO} and therefore for a certain range R_i although the tank volume would allow more fuel $m_{F,max}$. At this point B in the below figure additional flight range is only achieved by reducing the payload m_{PL} and by increasing the fuel mass accordingly up to a certain point C when the tank is full.
From that point a further range with a full tank is only possible by reducing the payload to zero. This point D is called the ferry range R_{Ferry}, as this is the maximum possible range for an aircraft with no payload and a full tank.

Table 4.1 Significant Payload-Range Coordinates

Point	Range-Coordinate	Payload-Coordinate
A	R_0= 0	Maximum payload m_{MPL}
B	Max. range with max. payload: R_{MPL}	Maximum payload m_{MPL}
C	Max. range with full tank: R_{MF}	Maximum fuel, full tank, m_{MF}
D	Max. range for no payload: R_{Ferry}	No payload, m_{PL}=0

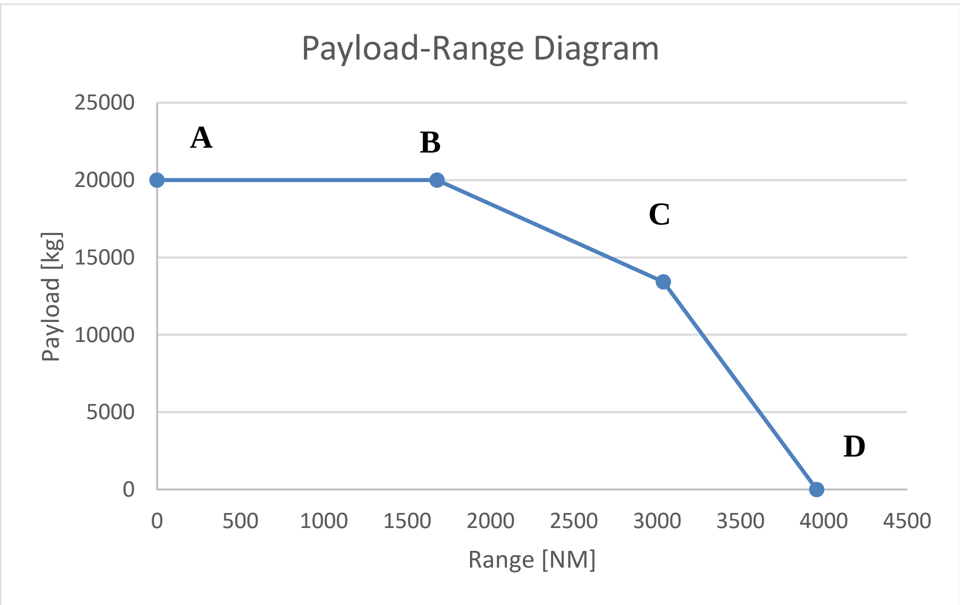


Figure 4.1 Payload-Range Diagram

4.2 Estimation of the Fuel Fraction

If the maximum payload m_{MPL} and the operating empty mass m_{OE} is known or calculated with the simple relationship

$$m_{MTO} = m_{OE} + m_{MPL} + m_F \tag{4.1}$$

we can estimate with the classical Breguet equation (2.7)

$$R = \frac{L}{D}\frac{V}{C_T.g} ln\left(\frac{m_i}{m_{i+1}}\right) \tag{4.2}$$

the range of point B in the Payload-range diagram. All we need is the aircraft mass at the beginning m_i and at the end m_{i+1} of the corresponding segment. The ratio m_i/m_{i+1} is a function of the consumed fuel $m_{F,i}$ and thus a function of the fuel fraction $M_{ff,i}$ for the flight segment i.

4.3 Mission Profile and Fuel Fraction

A typical mission profile (Figure 4.4) of an aircraft is consisting of different segments:
1. Taxi-Out 2. Take-Off 2. Climb, 3. Cruise, 4. Descent, 5. Landing 6. taxi-in). Further fuel is needed as reserve fuel m_{Res} in case of missed approach or for an alternate airport which is not considered in our below example.

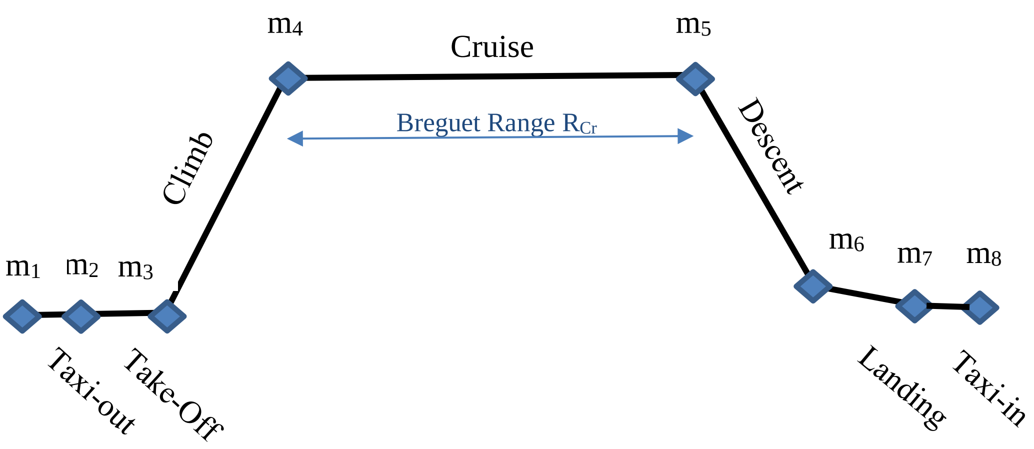


Figure 4.2: Typical mission profile

The ratio of the consumed fuel mass m_F for a complete flight mission relative to the take-off mass m_{TO} is

$$m_{ff} = \frac{m_F}{m_{TO}} \tag{4.3}$$

and hence the fuel fraction M_{ff}

$$M_{ff} = 1 - \frac{m_F}{m_{TO}} \tag{4.4}$$

The total fuel fraction M_{ff} is per definition the product (and not the sum!) of the fuel fractions $M_{ff,i}$ of each segment i:

$$M_{ff} = \prod_1^i M_{ff,i} \tag{4.5}$$

In our example with 7 different mission segments (i=7) the total fuel fraction is:

$$M_{ff} = M_{ff,1} \cdot M_{ff,2} \cdot M_{ff,3} \cdot M_{ff,4} \cdot M_{ff,5} \cdot M_{ff,6} \cdot M_{ff,7} \tag{4.6}$$

M_{ff} is relative to m_{TO} but the segment fuel fractions $M_{ff,i}$ are relative to the aircraft mass m_i at the beginning of the according flight phase i. With 7 segments in our example we have 8 defined points with a different aircraft mass m_i:

$$M_{ff} = \frac{m_2}{m_1} \cdot \frac{m_3}{m_2} \cdot \frac{m_4}{m_3} \cdot \frac{m_5}{m_4} \cdot \frac{m_6}{m_5} \cdot \frac{m_7}{m_6} \cdot \frac{m_8}{m_7} = \frac{m_8}{m_1} \tag{4.7}$$

So, the total mass fraction is equal to the mass ratio at the end and at the beginning of the mission. For each segment of the flight the following fuel fractions are given by statistics:

Table 4.2 Fuel Fraction for a typical flight

Mission segment		Fuel Fraction
1. Taxi-Out	$M_{ff,1}$	0.99
2. Take-Off	$M_{ff,2}$	0.995
3. Climb	$M_{ff,3}$	0.98
4. Cruise	$M_{ff,4}$	Calculated with Breguet
5. Descent	$M_{ff,5}$	0.99
6. Landing	$M_{ff,6}$	0.992
7. Taxi-In	$M_{ff,7}$	0.99
Total	$M_{ff} =$	$0.939 \cdot M_{ff,4}$

If we want to calculate the cruise range R_{CR} (Tab. 4.2) we only need the inverse mass ratio $m_i/m_{i+1} = m_{Cr,begin}/m_{Cr,end} = m_4/m_5$ of the cruise segment

$$\frac{m_i}{m_{i+1}} = \frac{1}{1-M_{ff,CR}} \tag{4.8}$$

to calculate the range:

$$R_{Cr} = \frac{L}{D}\frac{V}{c_T \cdot g} ln\left(\frac{m_{Cr,begin}}{m_{Cr,end}}\right) \tag{4.9}$$

Example:
L/D = 17,43
V = 242 m/s
C_T = 16,4 mg/N/s
$m_{Cr,begin}$ = 77000 kg, $m_{Cr,end}$ = 63000 kg,
=> $m_{Cr,begin}/m_{CR,end}$ = 77/63=1,222 => $M_{ff,Cr}$=63/77=0,811
Result: R_{Cr} = 5261 km = 2841 NM

To calculate the fuel fraction for a certain segment i we have to use the inversion of equation (3.x) but with a negative sign (!) in the exponent:

$$M_{ff,i} = \frac{m_{i+1}}{m_i} = e^{-\frac{R_i}{B_{S,i}}} \tag{4.10}$$

With the given range R_{Cr} from the payload-range diagram and the constant Breguet factor $B_{S,Cr}$ in our example we get the above given value for the fuel fraction $M_{ff,Cr}$ for the corresponding cruise segment.

Again, the Breguet factor B_S is based on constant parameters but only applicable for a certain segment because e.g. the airspeed is different for each mission phase.

Once the total fuel fraction M_{ff} is known, and thus the fuel mass ratio m_F/m_{MTO}, we are finally in the position to calculate the MTOW with the so called first law of aircraft design which is simply rearranged from Eqn. (xx):

$$m_{MTO} = \frac{m_{MPL}}{1-\frac{m_F}{m_{MTO}}-\frac{m_{OE}}{m_{MTO}}} \tag{4.11}$$

5 Aircraft Design Method

Under ecological aspect the aim of aircraft design can be described as the optimization of the Breguet aircraft range equation. To achieve this we have to fly with:

1. a maximum glide ratio E (L-over-D)
2. a low Specific Fuel Consumption (SFC)
3. a maximum fuel fraction M_{ff}
4. a high overall efficiency $\eta_{overall}$

For aircraft designer the most important challenge is to design a high aerodynamic airplane with the highest glide ratio $E=L/D=C_L/C_D$. Lift is mostly known or given, thus E depends only on drag D. The drag is consisting mainly of the parasite drag D_0, which depends on the skin friction (not related to lift) and the induced drag D_i which is produced by lift and primarily depending on the wing span b:

$$D = D_0 + D_i = D_0 + \frac{C_L^2}{A \cdot \pi \cdot e} \tag{5.1}$$

D [-] total drag of aircraft
D_0 [-] zero drag or parasite drag
D_i [-] induced drag, lift dependent
C_L [-] lift coefficient
A [-] aspect ratio, $A=b^2/S$, b= wing span [m], S= wing area [m²]
e [-] Oswald factor

For higher Mach number we have additionally consider the wave drag D_w and the interference drag D_{int}. In this case the total drag is composed by:

$$D = D_0 + D_i + D_w + D_{int} \tag{5.2}$$

Due to the significance of the drag and lift for aircraft design the drag and lift estimation is covered in a separate report.

Figure 5.1 will illustrate the above determinations and correlations in a graphic overview:

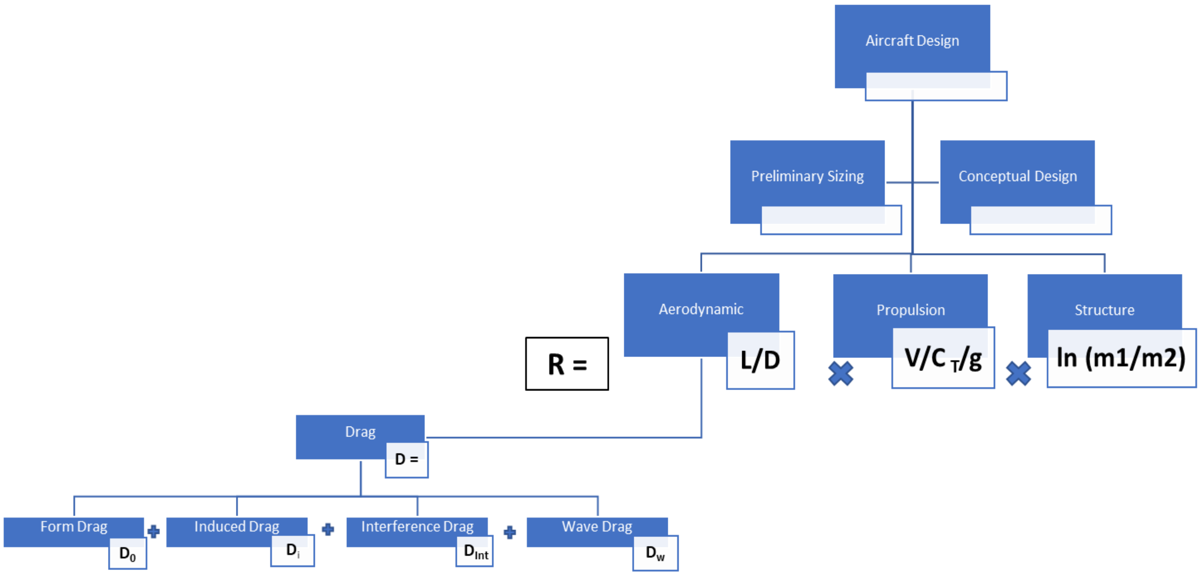


Figure 5.1 Aircraft Design correlation with Breguet Range Equation and Drag

5.1 Preliminary Sizing

If a certain performance of the aircraft is required (e.g. design range) we have to optimize the glide L/D, the specific fuel consumption SFC and the fuel fraction M_{ff}. All these parameters are depending on each other. If we change one parameter it will influence at the same time one or more other parameters. For example: By increasing the wing span in order to get a better aspect ratio and glide ratio the aircraft mass is increasing and probably the engine size with a higher trust and fuel consumption. This is exactly the snowball effect, which makes the estimation of optimized parameter a little bit complicate. But this is aircraft design.

To do so, one of the first step in aircraft design is the preliminary sizing of the aircraft considering the requirements and constraints which have to fulfil the according regulations (e.g. CS-25 or FAR Part 25 for jet) for each aircraft type. To determine the optimized design point, we need to plot a matching chart which shows for each certain flight manoeuvres the thrust-to weight ratio and the wing loading. Following flight manoeuvres will be computed and plotted with Eqns.(5.3)-(5.9):

- Take-off
- Climb (2nd segment)
- Climb (Missed approach)
- Landing
- Cruise

In Scholz 2015 a detailed process of preliminary sizing based on Loftin 1980 is described. The following modified figure 5.2 shows all steps in an overview:

Input		Output	
Statistic / Literature	**Requirements**	**Relative Values**	**Absolute values**
		Preliminary Sizing I	**Preliminary Sizing II**
		Matching Chart	**Payload-Range Diagram**

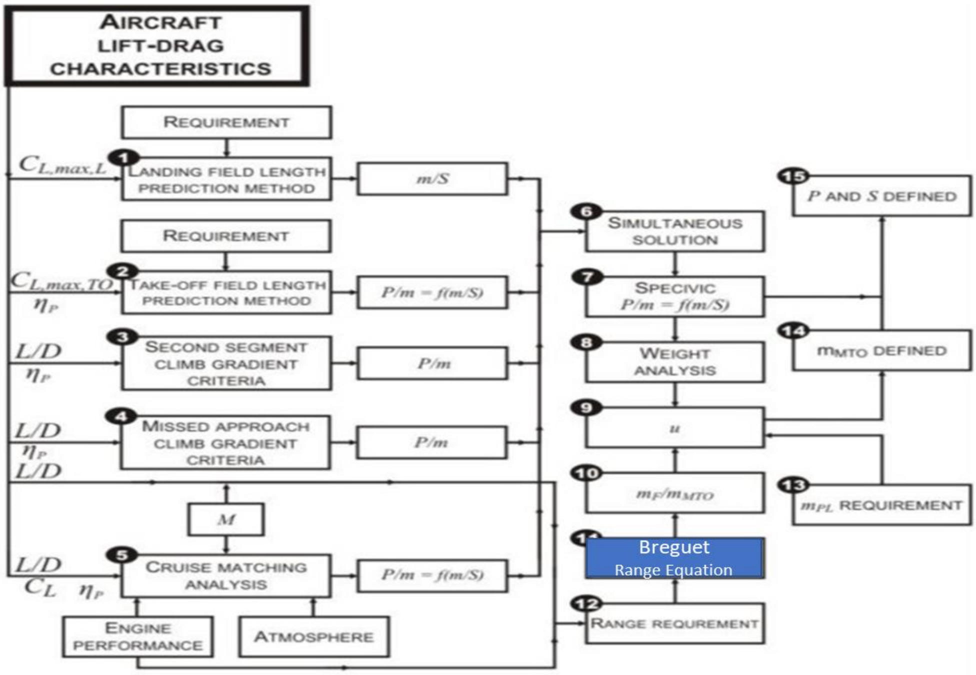


Figure 5.2 Preliminary sizing process for propeller aircraft based on Loftin 1980

For a jet aircraft the flow chart is the same. Only power P has to be exchanged with the thrust T and the matching chart has the thrust-to-weight ratio instead of power-to-mass ratio as a co-ordinate axis.

Following the above structure in Figure 5.3 the next subchapters will show the design process is subdivided into 3 main steps: the input data and requirements, the intermediate result of preliminary sizing I with relative values and the matching chart (constraint diagram) and the next step of preliminary sizing II with absolute values as a result calculated with the Breguet range equation and illustrated with the payload-range diagram.

5.1.1 Input Data

In order to do the first step of preliminary sizing I we have to introduce the top level aircraft requirements (TLAR) such as the CeRAS data which are very close to an Airbus A320 parameter and published on a website (CeRAS 2014). For redesigning the A320 as a reference aircraft these comprehensive data are suitable and covering different missions.

Table 5.1 Top Level Aircraft Requirements

Parameter	Symbol	Value (A320)	Remarks
Design range	R_D	2750 NM	
Maximum payload	m_{MPL}	20000 kg	Incl. 13608 kg for 150 Pax
Cruise Mach number	M_{Cr}	0.78	Max. operating speed: VMO=350m/s
Take-off field length	s_{TOFL}	< 2200 m	@sea level, ISA +15
Landing distance	s_{LFL}	< 1850 m	@ MLW, sea level, ISA, dry
Approach speed landing	V_{app}	< 138 m/s	
Wing span limit	b	< 36 m	

Further input data are needed for our Excel tool PreSTo (Preliminary Sizing Tool) which was developed at the HAW Hamburg in a classical version for a jet aircraft. Enhanced versions of PreSTo are also available for a jet and propeller driven aircraft with among others a more detailed calculations for the drag (NIŢĂ 2014) and the specific fuel consumption.

Table 5.2 Input data for PreSTo classic

Parameter	Symbol	Value (A320)	Remarks
Thrust T/O	T_{TO}	117,88 kN	@ sea level, M=0, ISA +15, 2xV2527-A5 engines
Thrust Specific fuel consumption TSFC	c_T	16,03 mg/N/s	@FL350, M_{CR}=0.78, ISA, without bleeded air off-takes
Operating empty mass ratio	m_{OE}/m_{MTO}	0,547	mOE=42092 kg, m_{MTO}=77000 kg

5.1.2 Relative Output Values (Preliminary Sizing I)

With the input data the thrust-to-weight ratio for different flight conditions (take-off, cruise and climb (2nd segment and missed approach) and the wing loading for landing are computed as follows:

Take-off:

$$a = \frac{\frac{T_{TO}}{m_{MTO} \cdot g}}{\frac{m_{MTO}}{S_W}} = \frac{k_{TO}}{s_{TOFL} \cdot \sigma \cdot C_{L,max,TO}} \tag{5.3}$$

where

a [-] Slope,
k_{TO} [-] Take-off Factor,
σ [-] Relative air density,
$C_{L,max,TO}$ [-] = 0.8· $C_{L,max,L}$ Maximum lift coefficient in take-off configuration,
S_{TOFL} [m] Take-off field length

Climb (2nd segment): $$\frac{T_{TO}}{m_{MTO} \cdot g} = \left(\frac{n_E}{n_E - 1}\right) \cdot \left(\frac{1}{E_{TO}} + sin\gamma\right) \quad (5.4)$$

where:

T_{TO} [N] Take-off thrust
n_E [-] Number of engines
E_{TO} [-] Glide ratio in take-off configuration
$sin\gamma$ [-] Climb gradient

Climb (Missed approach): $$\frac{T_{TO}}{m_{MTO} \cdot g} = \left(\frac{n_E}{n_E - 1}\right) \cdot \left(\frac{1}{E_L} + sin\gamma\right) \frac{m_{ML}}{m_{MTO}} \quad (5.5)$$

where:

T_{TO} [N] Take-off thrust
n_E [-] Number of engines
E_L [-] Glide ratio in landing configuration
$sin\gamma$ [-] Climb gradient

The wing loading for landing is a function of the landing distance with the maximum lift in landing configuration:

Landing: $$\frac{m_{MTO}}{S_W} = \frac{\frac{m_{ML}}{S_W}}{\frac{m_{ML}}{m_{MTO}}} \quad (5.6)$$

with $\frac{m_{ML}}{S_W} = k_L \cdot \sigma \cdot C_{L,max,L} \cdot s_{LFL}$ and $k_L = 0.03694 \cdot k_{APP}$

where:

k_L [-] Factor,
k_{APP} [-] Approach factor 1,70 m/s²
σ [-] Relative air density, is 1 at sea level
$C_{L,max,L}$ [-] Maximum lift coefficient in landing configuration
S_{LFL} [m] Landing field length, landing distance

For cruise we assume a horizontal unaccelerated flight with a certain constant Mach number but the aircraft speed v_{Cr} depends on the speed of sound a and therefore depends on the altitude h.or on the pressure p (h). The thrust in cruise $T_{Cr.}$ is a function of the bypass ratio BPR and the altitude h and the thrust ratio can be computed according Scholz 2015 as:

$$\frac{T_{Cr}}{T_{TO}} = (0{,}0013 \cdot BPR - 0{,}0397) \cdot h - 0{,}0248 \cdot BPR + 0{,}7125 \quad 5.7$$

The thrust-to weight ratio and the according wing loading are for cruise:

Cruise: $$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{1}{\left(T_{Cr}/T_{TO}\right) \cdot E} \quad 5.8$$

$$\frac{m_{MTO}}{S_W} = \frac{C_L \cdot M_{Cr} \cdot \gamma \cdot p}{g \cdot 2} \quad 5.9$$

where:

T_{TO} [N] Thrust at take-off
T_{Cr} [N] Thrust in cruise
h [km] Cruise altitude
p [[N/m²] Air pressure p=f(h)
E [-] glide ratio in cruise
C_L [-] Lift coefficient in cruise
M_{Cr} [-] Mach number in cruise
γ [-] =1.4 Ratio of specific air heat,

Eqn. (5.3) - (5.9) resulting in the curves of the below matching chart (Fig. 5.3)

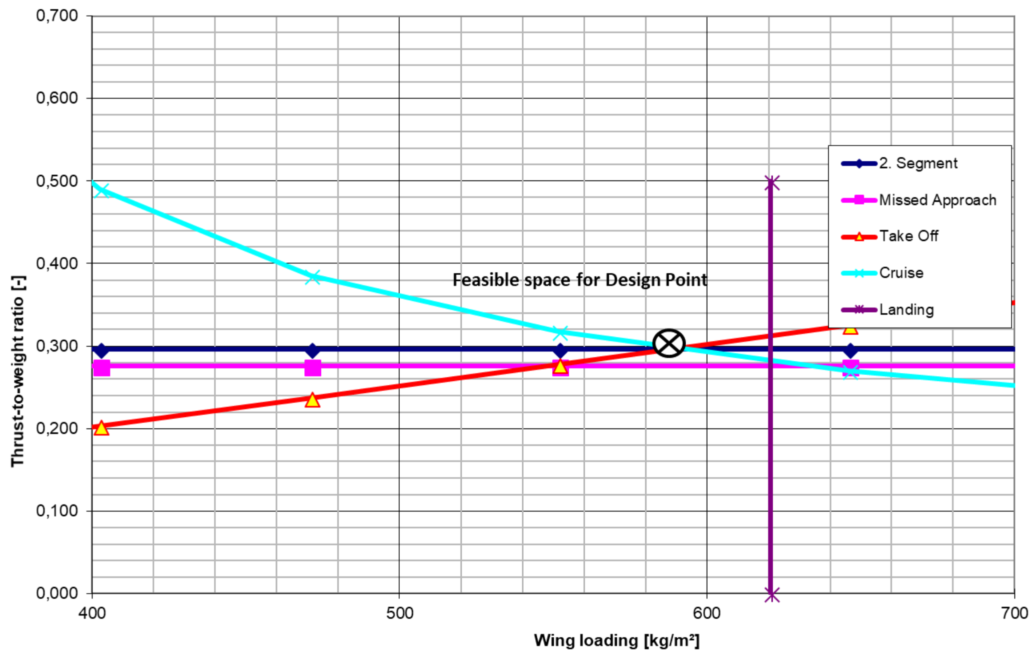


Figure 5.3 Matching chart with A320 data

To determine the design point, the first priority is to minimize the T/W ratio with a maximum wing loading.

5.1.2.1 Matching Chart Discussion

The chosen design point should have as much as possible the lowest thrust-to-weight ratio in the chart and must have a smaller wing loading than the calculated. This lead to the optimized design point.
Comparing e.g. the wing loading of the design point of the chart (Figure 5.4) with the realistic A320 data from CeRAS there are still differences (590 kg/m² and 630 kg/m2) for the wing loading.

The reason for the difference between the real design point of an A320 to our results is that the original version of this aircraft family was designed and optimized for a 73.5 t MTOW version with 111.2 kN thrust. For our sizing we are using an extended version with 77 t MTOW and more powered engines with 118 kN thrust. This results to the same thrust-to-weight ratio 0.308 but at the same landing distance S_{LFL} and the maximum lift coefficient for

landing $C_{L,max,L}$=2.8 and for take-off $C_{L,max,TO}$=2.2. The slope and curves in the matching chart are depending on these parameters and therefore lead to a different design point.

5.1.3 Absolut Output Values (Preliminary sizing II)

At the final stage of preliminary sizing II we are ready to calculate the absolute values for the aircraft size like the maximum take-off mass m_{MTO} (MTOW), the operating empty mass m_{OEW} (OEW), the fuel mass m_F, the wing area S_w and the required thrust T_{TO} for the engines. So far only relative values for the aircraft size are estimated. If we introduce now the maximum payload m_{MPL} from the TLAR requirement list (Tab 5.1) we get e.g. from the relative values of preliminary sizing I an absolute value for MTOW from Eqn. (4.11):

$$m_{MTO} = \frac{m_{MPL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}$$

where m_{MPL} is given from the requirements TLAR, the mass ratio m_{OE}/m_{MTO} is given by statistic (~ 0.55 or from an approximate equation) and the fuel mass ratio can be calculated as a function of

$$m_F/m_{MTO} = f(M_{ff}) = f(M_{ff,CR}) = f(Range, B_s)$$

Finally all further absolute sizing parameters like the operating empty mass m_{OE}, the fuel mass m_F, the wing area S_w, the take-off thrust T_{TO} and the maximum landing mass m_{ML} can now easily derived from the according relative sizing values (relative to m_{MTO}) of the designed aircraft (Eqn. 5.1 -5.4).

Again, the preliminary sizing is an iterative process. In the PreSTo sizing tool the main criterion for an additional iteration is the maximum landing mass m_{ML}. As long this value is lower than the sum of the zero fuel mass m_F and the reserve fuel mass $m_{F,Res}$

$$m_{ML} < m_{ZF} + m_{F,Res}$$

a new iteration is recommended by increasing the mass ratio m_{ML}/m_{MTO}.

5.1.3.1 Payload Range Diagram of Redesigned A320

When the preliminary sizing is finished, we have all data to draw the payload range diagram. In our sizing example (Appendix A) we redesigned an Airbus A320 aircraft with CeRAS data. All data for drawing the payload-range diagram are now available and shown in the below Fig. 5.4:

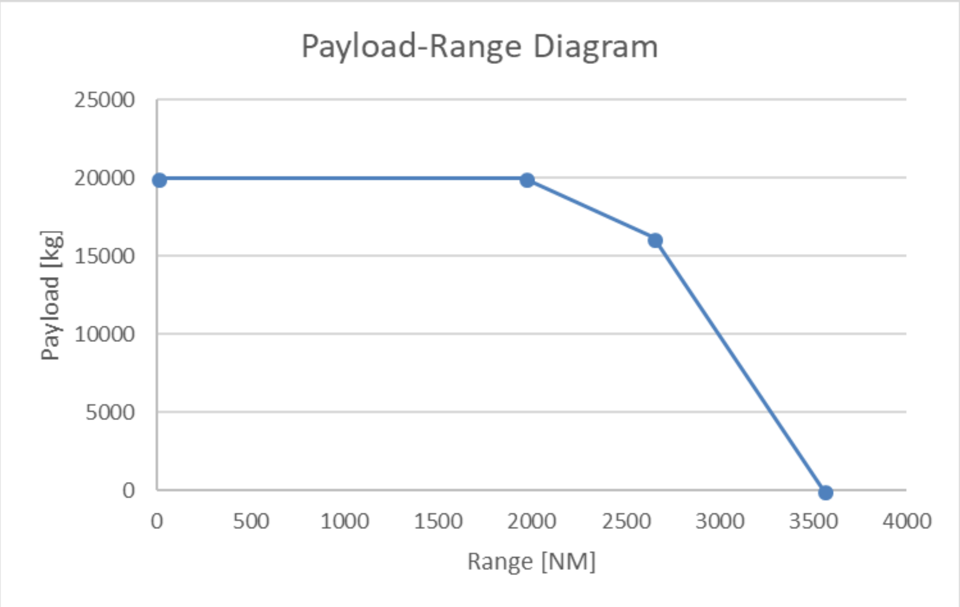


Figure 5.4 Payload-Range diagram of redesigned Airbus A320 with PreSTo

Comparing with the CeRAS payload-range diagrams (Appendix 2) there are small differences with the coordinates of points B and C as the next figure 5.6 is shown:

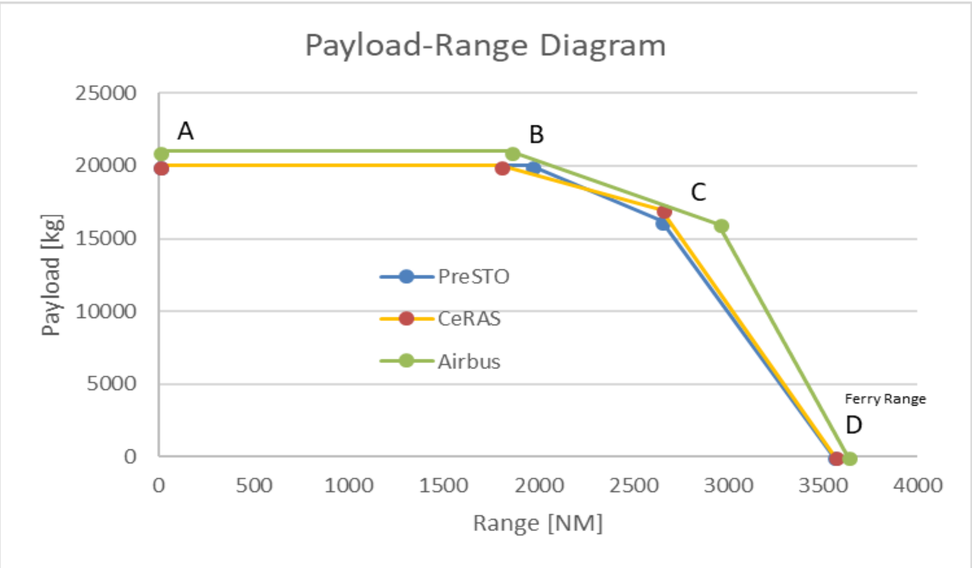


Figure 5.5 Payload-Range Diagram Airbus A320 from different sources

The difference to the Airbus payload diagram is based on the fact, that the diagram is created with a different engine (CFM56-5B), higher maximum payload (21000 kg) and considering a 2-step-cruise (35000/39000ft). As the slope between B and C are nearly the same, the fuel consumption SAR of both engine types is approximately equal.

Between CeRAS and PreSTo the slope difference between B and C is obviously much higher. The reason for that is the higher used SFC in PreSTo for the complete distance between A and C. In PreSTo only an average value of SFC (17,2 mg/N/s) is used and it will not distinguish between the SFC in climb, cruise and descent mode.

Table 5.3 Aircraft parameter from Airbus, CeRAS and PreSTo

Parameter	Airbus	CeRAS (Mission MTOW)	PreSTo	Deviation (PreSTo/CeRAS)
Engine type	2 x CFM56-5B	2x V2527-A5	V2527-A5	-
Thrust T_{TO}	2 x 120.1 kN	2x117.88 kN	2x118.05 kN	+ 0,1 %
TSFC c_T (with outtakes)	15.6 mg/N/s	16.73 mg/N/s (cruise)	17.17 mg/N/s (average)	+ 2.6%
Mach no. M_{CR}	0.76	0.78	0.78	0 %
Bypass ratio BPR	5.7	-	4.8	-
Wing area S_W	122.4 m²	122.41 m²	122,41 m²	0 %
Wing span b	34.1 m	34.1 m	34.1 m	0 %
Aspect ratio A	9.48	9.48	9.48	0 %
Glide ratio E	17.67	17.43	17.43	0 %
Mission Range R	-	2500 NM	2500 NM	0 %
Ferry Range	3630 NM	3560 NM	3556 NM	-0.1 %
MTOW m_{MTO}	77000 kg	77000 kg	77007 kg	+ 0.1 %
MLW m_{ML}	64500 kg	64500 kg	64686 kg	0.3%
MZFW m_{MZF}	60500 kg	62092 kg	62123 kg	+ 0.1 %
OEW m_{OE}	41244 kg	42092 kg	42123 kg	+ 0.1 %
Payload m_{MPL}	19256 kg	20000 kg	20000 kg	0 %
Max. Fuel m_{MF}	19159 kg	18678 kg	19272 kg	+ 3.1 %
Res. Fuel m_{Res}	4701 kg	3548 kg	3365 kg	-5.4 %
Wing loading	628 kg/m²	629.1 kg/m²	621 kg/m²	-1.3 %
Thrust-to-weight T/W	0.308	0.312	0.313	+ 0.3 %

5.1.3.2 Extended Payload-Range Diagram

For a better understanding and interpretation of the Breguet range equation and the fuel consumption for a standard flight we are introducing the extended payload-range diagram in Fig. 5.6, which additionally indicates the according absolute aircraft mass during the flight.

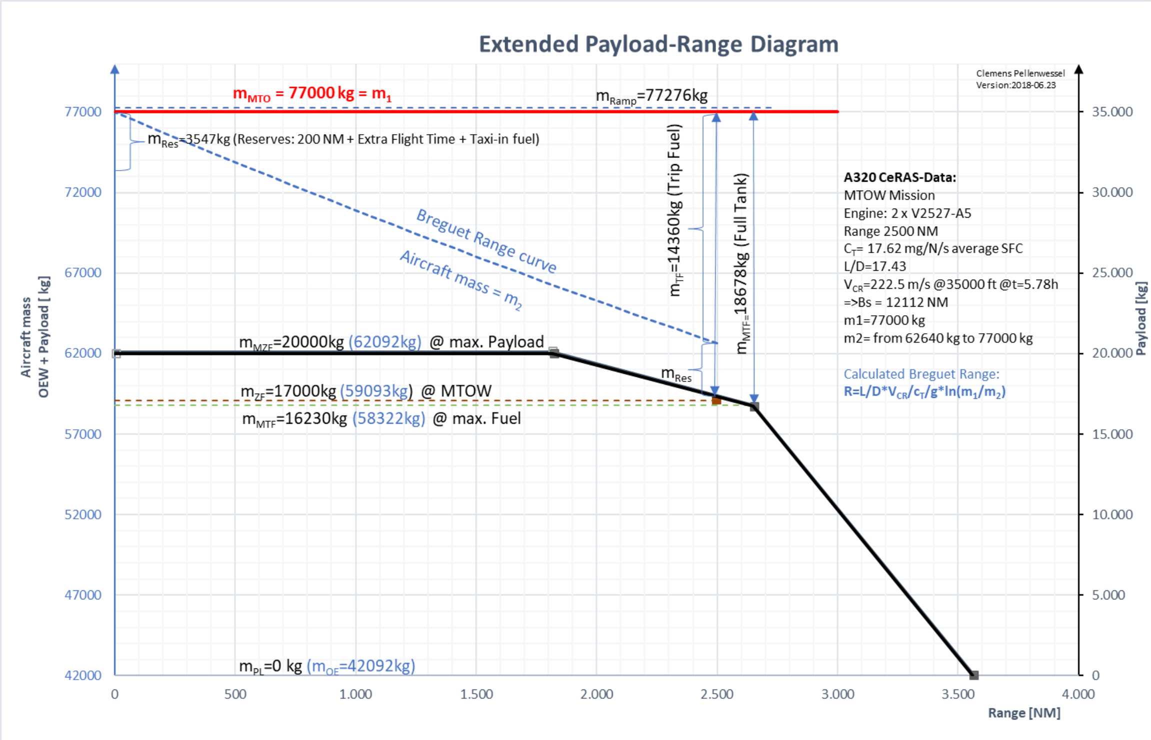


Figure 5.6 Extended Payload-Range Diagram for a A320 MTOW Mission

The initial mass m_1 (MTOW) and the final landing mass m_2, calculated with the Breguet range equation, are colourful highlighted in Fig. 5.6. The gap between both lines after a certain range is the mass of burned fuel. In consideration of the not used reserve fuel m_{Res}, both lines are not congruent but approximately parallel. Meaning the tangent of the Breguet curve and the SAR slope are about the same and therefor are both lines are useable to determine the fuel consumption.

6 Summary and Conclusion

With the extended payload-range diagram in Fig. 5.6 it is illustrated that the slope of the Breguet range curve (m_2-line) is approximately parallel to the SAR slope between point B and C only with the difference of the fuel reserve mRes. Beside that the Breguet range curve is more precise and the specific air range SAR (and therefore the fuel consumption) is from the mathematical point of view more precise computed as the first derivation R' of the Breguet range equation.

If it comes to the fuel consumption the Breguet range equation supply more precise results and is more exact for a certain range Ri than just the slope of the SAR from the payload-range diagram.

It has also been demonstrated that the Breguet range equation is a precise indicator of aircraft design and quality as it includes changes and improvements in aircraft design like the aerodynamic with the depending drag and lift, the fuel consumption of the engines and the aircraft structure and weight. To summerize this report: Aircraft design can be considered as an optimisation of the Breguet range equation.

For education in *Aircraft Design a First Approach With the Breguet Range Equation* is highly recommended and a practical method for understanding the complexity in aircraft design as it is described more detailed in Raymer 1992: *Aircraft Design: A Conceptual Approach*.

References

AIRBUS: A320 AIRCRAFT CHARACTERISTICS AIRPORT AND MAINTENANCE PLANNING AC, Blagnac Cedex 2005, URL: http://www.airbus.com/content/dam/corporate-topics/publications/backgrounders/techdata/aircraft_characteristics/Airbus-Commercial-Aircraft-AC-A320-Feb18.pdf archived as: https://bit.ly/2MWuOiM

AIRBUS: Airbus Rolls Royce and Siemens team up for electric future Partnership launches E-Fan X hybrid-electric flight demonstrator, Newsroom 18.11.2017.
URL: https://www.airbus.com/newsroom/press-releases/en/2017/11/airbus--rolls-royce--and-siemens-team-up-for-electric-future-par . archived as: https://bit.ly/2K6rqUNj

CeRAS, Central Reference Aircraft data System, Aachen 2014, URL: http://ceras.ilr.rwth-aachen.de/trac/wiki/CeRAS/AircraftDesigns/CSR01 archived as: https://bit.ly/2lyhAwn

HARTMANN, Johannes ; PFEIFFNER, Till: NASA/DLR Design Challenge 2018 - *Entwirf das Flugzeug der Zukunft*; Deutsches Zentrum für Luft- und Raumfahrt (DLR) ; Braunschweig2018;archived as http://bit.ly/2GLmTBs .

HEPPERLE, MARTIN; *Electric Flight – Potential and Limitations*; DLR Institute of Aerodynamics and Flow Technology; Braunschweig 2013

LOFTIN, LAURENCE K.; *Subsonic Aircraft: Evolution and the Matching of Size for Performance*, NASA Reference Publication 1060, Hampton, Virginia 1980,
URL: https://ia801208.us.archive.org/29/items/NASA_NTRS_Archive_19800020744/NASA_NTRS_Archive_19800020744.pdf

NASA; NASA Strategic Plan; *ARMD Strategic Thrust 3: Ultra-Efficient Commercial Vehicles Subsonic Transport*, Washington DC *2017; URL:*https://www.nasa.gov/sites/default/files/atoms/files/sip-2017-03-23-17-high.pdf, archieved as https://go.nasa.gov/2MnHBtK ;

NIŢĂ, MIHAELA; SCHOLZ, DIETER; German Aerospace Congress 2012; *Estimating the Oswald factor from basic aircraft geometrical parameter* , Hamburg 2012; URL:http://www.profscholz.de/

PELLENWESSEL, CLEMENS; SCHMIDT, JULIAN: *Entwurf eines Bausatzflugzeugs eMotion*, Flugzeug-Design-Wettbewerb der Oskar Ursinus Vereinigung OUV, Hamburg 2012, URL: http://www.ouv.de/fly/wp-content/themes/awake/styles/ouv/pdf/dw_2012_06_1.pdf archived as: https://bit.ly/2trFYDZ

Raymer, Daniel P.; *Aircraft design: a conceptual approach*, American Institute of Aeronautics and Astronautics Education Series, 2nd Edition, Washington DC 1992, ISBN 0-930403-51-7

SCHOLZ, DIETER: *Lecture Notes Aircraft Design*, HAW Hamburg University of Applied Sciences, Hamburg 2015; URL: http://www.fe.profscholz.de

SCHOLZ, DIETER, *Preliminary Sizing Tool PreSTo*, Excel spreadsheet, Hamburg 2017, URL: http://www.fzt.haw-hamburg.de/pers/Scholz/materialFE/A-C_Preliminary_Sizing.xls

WIKIPEDIA - THE FREE ENCYCLOPEDIA: *IAE V2500*, 2018, – URL: https://en.wikipedia.org/wiki/IAE_V2500 (2018-06-25)

YOUNG, TREVOR: *Lecture Notes Flight Mechanics*, University of Limerick, Limerick 2001, URL: http://www.fm.profscholz.de

Appendix A

PreSTo spreadsheets

1.) Preliminary Sizing I								**CeRAS-Data**
Calculations for flight phases approach, landing, tak-off, 2nd segment and missed approach								
Bold blue values represent input data.				Author:				
Values based on experience are light blue. Usually you should not change these values!				**Prof. Dr.-Ing. Dieter Scholz, MSME**				
Results are marked **red**. Don't change these cells!				**HAW Hamburg**				
Interim values, constants, ... are in black!				http://www.ProfScholz.de				
"<<<<" marks special input or user action.				Example data: Ceras A320-200				
				Status: 2018-07-03, Clemens Pellenwessel				
Approach								
Factor	k_{APP}	1,70	$(m/s^2)^{0.5}$					
Conversion factor		1,944	kt / m/s					
Given: landing field length		**no**		**<<<< Choose according to task** (ja = yes; nein = no)				
Landing field length	s_{LFL}	**1513**	**m**					
Approach speed	V_{APP}	66,2	m/s					
Approach speed	V_{APP}	**128,7**	**kt**	$V_{APP} = k_{APP}\cdot\sqrt{s_{LFL}}$				
Given: approach speed		**yes**						
Approach speed	V_{APP}	**138,0**	**kt**	$s_{LFL} = \left(\frac{V_{APP}}{k_{APP}}\right)^2$				CeRAS: Vapp,Ldg=138 kt
Approach speed	V_{APP}	71,0	m/s					
Landing field length	s_{LFL}	**1740**	**m**					
Landing								
Landing field length	s_{LFL}	1740	m					
Temperature above ISA (288,15K)	ΔT_L	**0**	**K**					CeRAS: ISA
Relative density	σ	1,000						
Factor	k_L	0,107	kg/m³	$k_L = 0{,}03694\, k_{APP}^2$				
Max. lift coefficient, landing	$C_{L,max,L}$	**2,8**						CeRAS: Cl,max,ldg=2,80
Mass ratio, landing - take-off	m_{ML} / m_{TO}	**0,84**		$m_{ML}/S_W = k_L\cdot\sigma\cdot C_{L,max,L}\cdot s_{LFL}$				CeRAS: 0,84
Wing loading at max. landing mass	m_{ML} / S_W	**521**	**kg/m²**					
Wing loading at max. take-off mass	m_{MTO} / S_W	**621**	**kg/m²**	$m_{MTO}/S_W = \frac{m_{ML}/S_W}{m_{ML}/m_{MTO}}$				CeRAS: 629,1
Take-off								
38	s_{TOFL}	**2184,2**	**m**					CeRAS: TODR=2184,2m
Temperatur above ISA (288,15K)	ΔT_{TO}	**15**	**K**					CeRAS: ISA+15
Relative density	σ	0,951						
Factor	k_{TO}	2,3	m³/kg	changed from 2,34				
Exprience value for $C_{L,max,TO}$	0,8 * $C_{L,max,L}$	2,24		$a = \frac{T_{TO}/(m_{MTO}\cdot g)}{m_{MTO}/S_W} = \frac{k_{TO}}{s_{TOFL}\cdot\sigma\cdot C_{L,max,TO}}$				
Max. lift coefficient, take-off	$C_{L,max,TO}$	**2,2**						CeRAS: CL,max,TO= 2,2
Slope	a	**0,0005036**	**kg/m³**					
Thrust-to-weight ratio	T_{TO}/m_{MTO}*g at m_{MTO}/S_W calculated from landing	**0,313**						CeRAS: 0,312
2nd Segment								
Calculation of glide ratio								
Aspect ratio	A	**9,48**						CeRAS: A=9,48
Lift coefficient, take-off	$C_{L,TO}$	1,53						
Lift-independent drag coefficient, clean	$C_{D,0}$ (for calculation: 2. Segment)	0,019			n_E	sin(γ)		CeRas: CD,visc=0,0188
Lift-independent drag coefficient, flaps	$\Delta C_{D,flap}$	0,021	???		2	0,024		
Lift-independent drag coefficient, slats	$\Delta C_{D,slat}$	0,000			3	0,027		
Profile drag coefficient	$C_{D,P}$	0,040			4	0,030		
Oswald efficiency factor; landing configuration	e_TO	0,525						SCHOLZ / NITA A320: e_CR= e_TO=e_CR*0,7
Glide ratio in take-off configuration	E_{TO}	8,06						
Calculation of thrust-to-weight ratio								
Number of engines	n_E	**2**						
Climb gradient	sin(γ)	0,024		$\frac{T_{TO}}{m_{MTO}\cdot g} = \left(\frac{n_E}{n_E-1}\right)\cdot\left(\frac{1}{E_{TO}}+\sin\gamma\right)$				
Thrust-to-weight ratio	T_{TO} / m_{MTO}*g	**0,296**						
Missed approach								
Calculation of the glide ratio								
Lift coefficient, landing	$C_{L,L}$	1,66				CS-25	FAR Part 25	
Lift-independent drag coefficient, clean	$C_{D,0}$ (for calculation: Missed Approach)	0,019		$\Delta C_{D,gear}$		0,000	0,015	
Lift-independent drag coefficient, flaps	$\Delta C_{D,flap}$	0,028						
Lift-independent drag coefficient, slats	$\Delta C_{D,slat}$	0,000						
Choose: Certification basis	CS-25	**no**		**<<<< Choose according to task**				
	FAR Part 25	**yes**						
Lift-independent drag coefficient, landing gear	$\Delta C_{D,gear}$	0,015		n_E	sin(γ)			
Profile drag coefficient	$C_{D,P}$	0,062		2	0,021			
Glide ratio in landing configuration	E_L	6,98		3	0,024			
				4	0,027			
Calculation of thrust-to-weight ratio								
Climb gradient	sin(γ)	0,021		$\frac{T_{TO}}{m_{MTO}\cdot g} = \left(\frac{n_E}{n_E-1}\right)\cdot\left(\frac{1}{E_L}+\sin\gamma\right)\cdot\frac{m_{ML}}{m_{MTO}}$				
Thrust-to-weight ratio	T_{TO} / m_{MTO}*g	**0,276**						

3.) Preliminary Sizing II

Calculations for cruise, matching chart, fuel mass, operating empty mass and aircraft parameters m_{MTO}, m_L, m_{OE}, S_W, T_{TO}, ...

Parameter		Value		Parameter	Value		1,316074013	
By-pass ratio	BPR	4,8	V2527-A5 engine	V/V_{md}	1	Jet, Theory, Optimum:	1,316074013	changed to 1 => E=Emax
Max. glide ratio, cruise	E_{max}	17,43	(aus Teil 2)	$C_L/C_{L,md}$	1,000			CeRAS: 17,43
Aspect ratio	A	9,48	(aus Teil 1)	C_L	0,658			CeRAS: 9,48
Oswald eff. factor, clean	e	0,77		E	17,430			SCHOLZ/NITA 2012 e=0,77 @M=0.78
Zero-lift drag coefficient	$C_{D,0}$	0,0189						CeRAS 0,188
Lift coefficient at E_{max}	$C_{L,md}$	0,66						
Mach number, cruise	M_{CR}	0,78						CeRAS M=0,78

$$C_{D,0} = \frac{\pi \cdot A \cdot e}{4 \cdot E_{max}^2}$$

$$C_{L,md} = \sqrt{C_{D,0} \cdot \pi \cdot A \cdot e}$$

$$C_L / C_{L,md} = 1/(V/V_{md})^2$$

$$E = E_{max} \cdot \frac{2}{\frac{1}{\left(\frac{C_l}{C_{l,md}}\right)} + \left(\frac{C_l}{C_{l,md}}\right)}$$

Constants			
Ratio of specific heats, air	γ	1,4	
Earth acceleration	g	9,81	m/s²
Air pressure, ISA, standard	p_0	101325	Pa
Euler number	e	2,7182818	

$$\frac{T_{TO}}{m_{MTO} \cdot g} = \frac{1}{(T_{CR}/T_{TO}) \cdot E}$$

$$\frac{m_{MTO}}{S_W} = \frac{C_L \cdot M^2}{g} \cdot \frac{\gamma}{2} \cdot p(h)$$

Altitude		Cruise				2nd Segment	Missed appr.	Take-off	Cruise	Landing
h [km]	h [ft]	T_{CR}/T_{TO}	T_{TO}/m_{MTO}*g	p(h) [Pa]	m_{MTO}/S_W [kg/m²]	T_{TO}/m_{MTO}*g	T_{TO}/m_{MTO}*g	T_{TO}/m_{MTO}*g	T_{TO}/m_{MTO}*g	T_{TO}/m_{MTO}*g
0	0	0,593	0,097	101325	2894	0,296	0,276	1,46	0,10	
1	3.281	0,560	0,102	89873	2567	0,296	0,276	1,29	0,10	
2	6.562	0,527	0,109	79493	2270	0,296	0,276	1,14	0,11	
3	9.843	0,493	0,116	70105	2002	0,296	0,276	1,01	0,12	
4	13.123	0,460	0,125	61636	1760	0,296	0,276	0,89	0,12	
5	16.404	0,426	0,135	54015	1543	0,296	0,276	0,78	0,13	
6	19.685	0,393	0,146	47176	1347	0,296	0,276	0,68	0,15	
7	22.966	0,359	0,160	41056	1173	0,296	0,276	0,59	0,16	
8	26.247	0,326	0,176	35595	1017	0,296	0,276	0,51	0,18	
9	29.528	0,292	0,196	30737	878	0,296	0,276	0,44	0,20	
10	32.808	0,259	0,222	26431	755	0,296	0,276	0,38	0,22	
11	36.089	0,225	0,255	22627	646	0,296	0,276	0,33	0,25	
12	39.370	0,192	0,299	19316	552	0,296	0,276	0,28	0,30	
13	42.651	0,158	0,362	16498	471	0,296	0,276	0,24	0,36	
14	45.932	0,125	0,459	14091	402	0,296	0,276	0,20	0,46	
15	49.213	0,092	0,627	12035	344	0,296	0,276	0,17	0,63	
					621					0
					621					0,5
Remarks:	1m=3,281 ft	T_{CR}/T_{TO}= f(BPR,h)	Gl.(5.27)	Gl. (5.32/5.33)	Gl. (5.34)	from sheet 1.)	from sheet 1.)	from sheet 1.)	Repeat for plot	from sheet 1.)

Wing loading	m_{MTO}/S_W	621	kg/m²	**<<<< Read design point from matching chart!**	CeRAS: 629,1
Thrust-to-weight ratio	$T_{TO}/(m_{MTO}$*g)	0,313		<<<< Given data is correct when take-off and landing is sizing the aircraft at the same time.	CeRAS: 0,312
Thrust ratio	$(T_{CR}/T_{TO})_{CR}$	0,184			
Conversion factor	m -> ft	0,305	m/ft		
Cruise altitude	h_{CR}	12250	m	calculated h_{CR} =f(BPR, T/W, L/D)	
Cruise altitude	h_{CR}	40191	ft		
Temperature, troposphere	$T_{Troposphäre}$	208,52	K	$T_{stratosphäre}$ [K] 216,65	
Temperature, h_{CR}	$T(h_{CR})$	216,65			
Speed of sound, h_{CR}	a	295	m/s	M= 0,78	
Cruise speed	V_{CR}	223	m/s	V=a*M average air speed from T/O to landing	CeRAS: given time 5.784 h 222,51
Conversion factor	NM -> m	1852	m/NM		
Design range	R	2500	NM		CeRAS: MTOW Range= 2500 NM
Design range	R	4630000	m	R_Cruise [m]= 4176260	R_Cruise [NM] 2255 changed until N
Distance to alternate	$s_{to\_alternate}$	200	NM		
Distance to alternate	$s_{to\_alternate}$	370400	m	**Reserve flight distance:**	CeRAS: 200NM
Chose: FAR Part121-Reserv	domestic	no			CeRAS: JAR-OPS 1.255
	international	yes			CeRAS: 200NM=370400m
Extra-fuel for long range		5%			CeRAS: 5% included???
Extra flight distance	s_{res}	601900	m		
Spec.fuel consumption, cruis	SFC_{CR}	1,76E-05	kg/N/s	typical value 1,60E-05 kg/N/s	CeRAS: SFC 16,73 mg/N/s with bleeded air
				Extra time:	CeRAS: SFC average = 17,62 mg/N/s (climb including reserve fuel
Breguet-Factor, cruise	B_s	22437431	m		
Fuel-Fraction, cruise	$M_{ff,CR}$	0,830			Eqn. for Fuel Fraction , Cruise Mff,Re correc
Fuel-Fraction, extra flight dist	$M_{ff,RES}$	0,974			

FAR Part 121	s_{res}	
domestic	370400	m
international	601900	m

FAR Part 121	t_{loiter}	
domestic	2700	s
international	1800	s

$$B_S = \frac{L}{D} \cdot \frac{V}{R \cdot g} = const.$$

$$M_{ff,i} = \frac{m_{i+1}}{m_i} = e^{-\frac{R_i}{B_{S,i}}}$$

Loiter time	t_{loiter}	1800	s		Trip fuel	m_{TF}	14360
Spec.fuel consumption, loiter	SFC_{loiter}	1,76E-05	kg/N/s		Reserve fuel	$m_{F,Res}$	3394
Breguet-Factor, flight time	B_t	100838	s		Taxi out fuel	$m_{F,Taxiout}$	276
Fuel-Fraction, loiter	$M_{ff,loiter}$	0,982			Taxi in fuel	$m_{F,Taxiin}$	153
					CeRAS:		
					$m_{F,mission}$	18183	
Fuel-Fraction, engine start	$M_{ff,engine\ start}$	1,000	**<<<< Copy**		taxi out & engir	0,985	276
Fuel-Fraction, taxi	$M_{ff,taxi\ out}$	0,985	**<<<< values**		taxi in	0,992	153
Fuel-Fraction, take-off	$M_{ff,TO}$	1,000	**<<<< from**				
Fuel-Fraction, climb	$M_{ff,CLB}$	0,975	**<<<< table**			0,975	
Fuel-Fraction, descent	$M_{ff,DES}$	1,000	**<<<< on the**				
Fuel-Fraction, landing	$M_{ff,L}$	0,992	**<<<< right !**				

Phase	M_{ff} per flight phases [Roskam]	
	transport jet	business jet
engine start	0,990	0,990
taxi out	0,990	**0,995**
take-off	0,995	0,995
climb	0,980	0,980
descent	0,990	0,990
landing	0,992	0,992

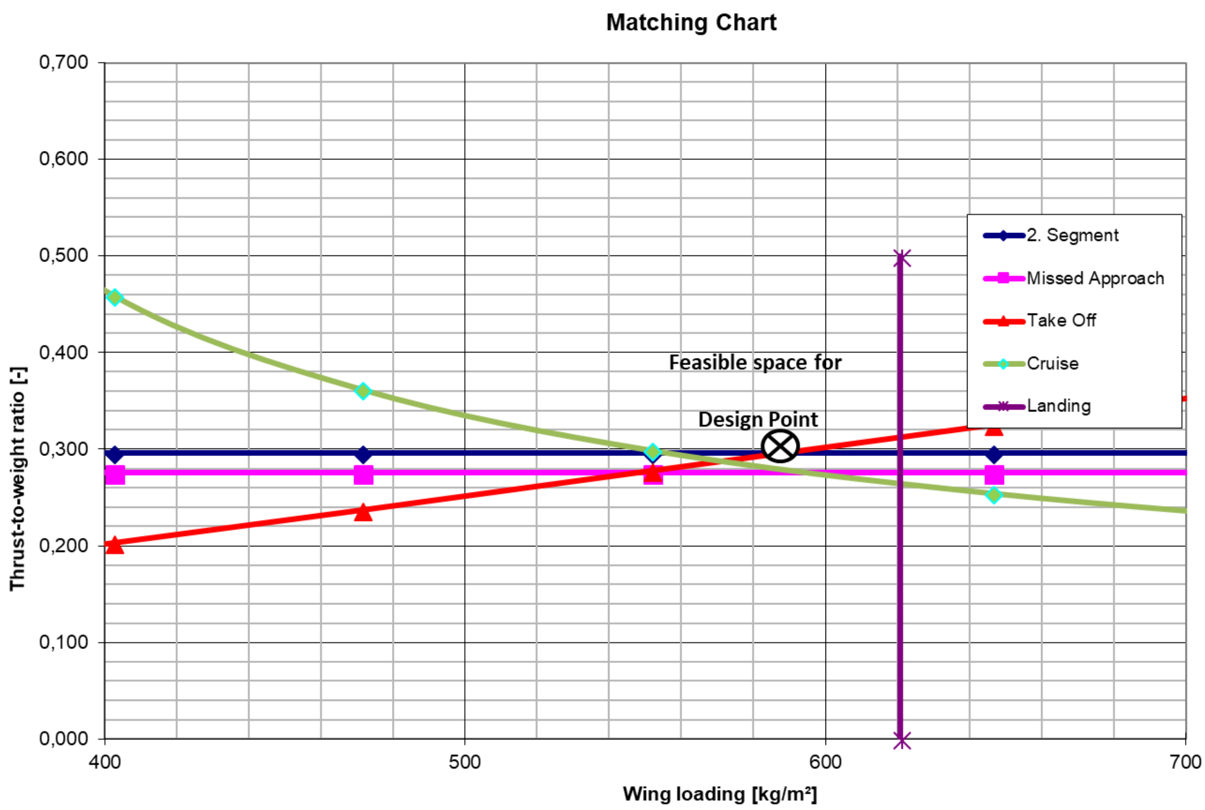
Fuel-Fraction, standard flight	$M_{ff,std}$	0,803		Trip fuel fraction	
Fuel-Fraction, all reserves	$M_{ff,res}$	0,956		new equation without Mff,climb and Mff,descent	
Fuel-Fraction, total	M_{ff}	0,768			CeRAS: 0,757
Mission fuel fraction	m_F/m_{MTO}	0,232			CeRAS: 0,243
Realtive operating empty mas	m_{OE}/m_{MTO}	0,555		acc. to Loftin	
Realtive operating empty mas	m_{OE}/m_{MTO}	0,547		from statistics (if given)	CeRAS: 0,547
Realtive operating empty mas	m_{OE}/m_{MTO}	0,547		**<<<< Choose according to task**	
Choose: type of a/c	short / medium range	yes		**<<<< Choose according to task**	
	long range	no			
Mass: Passengers, including	m_{PAX}	93,0	kg		
Number of passengers	n_{PAX}	150			CeRAS: 90,72 kg
Cargo mass	m_{cargo}	3050	kg		
Payload	m_{PL}	17000	kg		CeRAS: 17000kg

in kg	Short- and Medium Range	Long Range
m_{PAX}	93,0	97,5

$$m_{MTO} = \frac{m_{MPL}}{1 - \frac{m_F}{m_{MTO}} - \frac{m_{OE}}{m_{MTO}}}$$

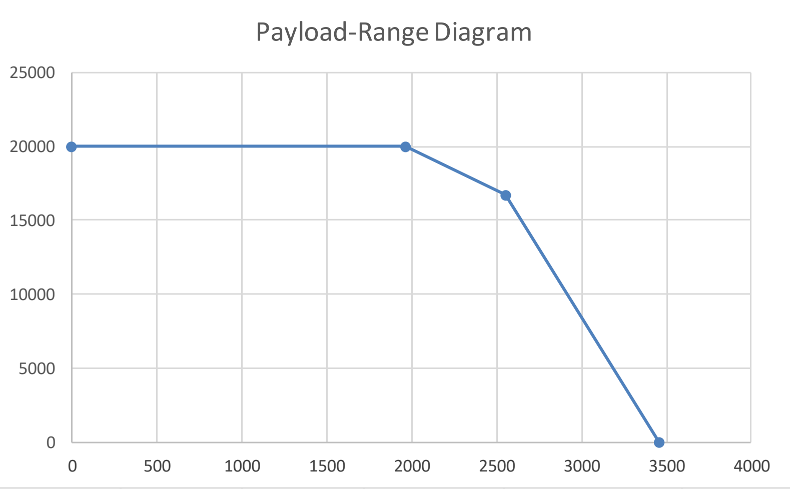
Max. Take-off mass	m_{MTO}	76999	kg		CeRAS: 77000kg
Max. landing mass	m_{ML}	64679	kg		CeRAS: 64500kg
Operating empty mass	m_{OE}	42118	kg		CeRAS: 42092kg
Mission fuel fraction, standar	m_F	17880	kg	Mission fuel, not fraction!!!	CeRAS: 18183kg
Wing area	S_W	124,1	m²		CeRAS: 122.4 m²
Take-off thrust	T_{TO}	236085	N	**all engines together**	
T-O thrust of ONE engine	T_{TO}/n_E	118043	N	**one engine**	
T-O thrust of ONE engine	T_{TO}/n_E	26536	lb	**one engine**	
Fuel mass, needed	$m_{F,erf}$	18778	kg	**Trip fuel $m_{F,Tri}$** 14680	CERAS: 18668kg
Fuel density	ρ_F	800	kg/m³		
Fuel volume, needed	$V_{F,erf}$	23,5	m³	(check with tank geometry later on)	
Max. Payload	m_{MPL}	20000	kg		CeRAS: 20000kg
Max. zero-fuel mass	m_{MZF}	62118	kg		CeRAS: 62092kg
Zero-fuel mass	m_{ZF}	59118	kg		CeRAS: 59012kg
Fuel mass, all reserves	$m_{F,res}$	3364	kg		CeRAS: 3385kg
Check of assumptions	check:	m_{ML}		> $m_{ZF} + m_{F,res}$	
		64679	kg	> 62483 kg	
				yes	
				Aircraft sizing finished!	

2.) Max. Glide Ratio in Curise				
Estimation of k_E by means of 1.), 2.) or 3.)				
1.) From theory				
Oswald efficiency factor for k_E	e	0,75		from NITA 2012 A320: e_CR= 0,75
Equivalent surface friction coefficient	$C_{f,eqv}$	0,003		
Factor	k_E	14,0		
2.) Acc. to RAYMER				
Factor	k_E	15,8		
3.) From own statistics				
Factor	k_E	???		
Estimation of max. glide ratio in cruise, E_{max}				
Factor	k_E chosen	14,0		**<<<< Choose according to task**
Relative wetted area	S_{wet} / S_w	6,1		S_{wet} / S_w = 6,0 ... 6,2
Aspect ratio	A	9,48	(from sheet 1)	
Max. glide ratio	E_{max}	17,47		
	or			
Max. glide ratio	$E_{max\ chosen}$	17,43	CeRAS=17,4:	**<<<< Choose according to task**



Payload-Range Diagram Coordinates:

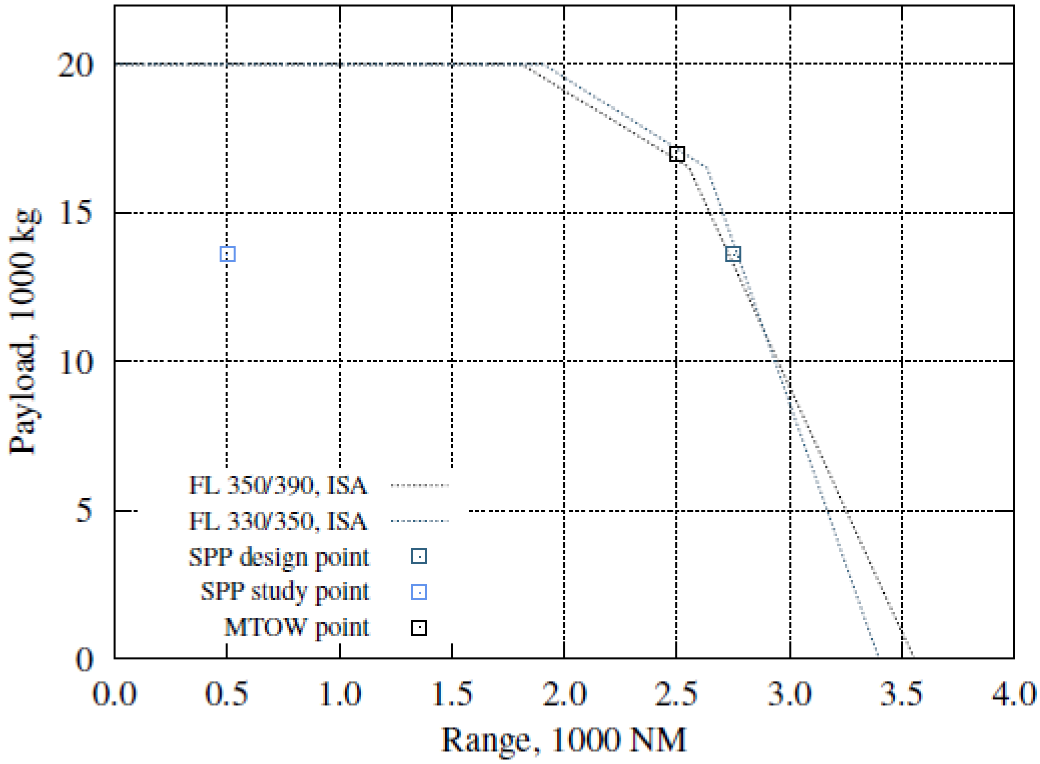
		Mass aircraft	Range [NM]	Mass payload
	MTOW	76999	0	34880
	MTOW	76999	4000	34880
Point A	MZFW	62118	0	20000
Point B	MZFW	62118	1962	20000
Point C		58821	2552	16703
Point D		42118	3455	0



Fundamentals:		
g=	9,81	m/s²
L_over_D	17,43	
$V_{average}$	222,5	m/s
SFC =	0,00001762	g/kN/s
$m_{F,res}$=	3547	kg
$m_{F,MT}$=	18178	kg
$B_{S1,A\text{-}C}$=	12115	NM
Point B:		
m2=MZFW+mF,re	65483	kg
R_B=	1962	NM
Point C (Rmax):		
Trip fuel =	14631	kg
m1=MTOW =	76999	kg
m2=	62368	kg
Bs=	12109	NM
R_{max}=	2552	NM
Point D (Ferry Range):		
m2=mOE+mRes	45666	kg
m1=	60296	kg
$V_{average,Ferry}$	228,3	m/s
$B_{S,Ferry}$	12433	NM
R_{Ferry} =	3455	NM

Appendix B

B.1 CeRAS A320 Payload-Range Diagram



B.2 Airbus A320 Payload-Range Diagram